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What is quantum computing?

Seminar Paper

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Foreword

This paper was created in the terms of a seminar about post-quantum cryptography without prior academic knowledge of quantum mechanics and technology. In order to use advanced terms I created a glossary at the end of this document that explains most of the used scientific terms to the extent needed to understand this paper. Knowledge of basic quantum mechanics aswell as common practices in notation (bra-ket-notation / Dirac notation) and mathematical concepts (e.g. vectors) is recommended and for some parts (especially formulas) required.

1. What is quantum computing?

1.1. Minimal Requirements

In his paper *TOPICS IN QUANTUM COMPUTERS* (1996) David DiVincenzo provides a list of minimal requirements to create a general purpose quantum computer (GPQC) (opposing to a quantum system in general). A quantum system is defined as a computing system that utilizes quantum effects while a GPQC is a quantum system that is capable of executing any permutation over a given set of operations to fulfill virtually any purpose (DiVincenzo, 1996).

David DiVincenzo is a theoretical physicist who is seen as one of the pioneers in the area of quantum computer. During his research at IBM he published the paper noticed above and a year later, in 1997, he proposed the Loss-DiVincenzo quantum computer [QCQD] which fulfills the criteria he provided himself (Loss & DiVincenzo, 1998).

The requirements DiVincenzo proposed are as follows:

- Determinism

The underlying quantum system should be possible to be described by a Hilbert space. An additional requirement to this Hilbert space is a precise determinism about its state. DiVincenzo states that making „only statistical statements […] won't do".

It is also recommended that it should be possible to represent the quantum system as a direct product form, resulting in a description mapping the state of each part of the system to the respective part.
This might result in Hilbert spaces with a huge amount of dimensions.
In his paper DiVincenzo provides an example of 49 elements, each having a certain spin state with 5 possible values, resulting in a Hilbert space with 5^{49} dimensions.
If a quantum system is described by a two-dimensional Hilbert space it is termed a quantum bit or *qubit*.

- Possibility of a ground state

This requirement describes the necessity of being able to place the underlying quantum system into a reproducible ground state (or: base state, starting state). DiVincenzo provides the example of „all spins down" for this. This is required to achieve determinism of computation.

- Isolation

An ideal GPQC would be completely isolated from its environment. Since that is unachievable and would also negate the purpose of such a device, the goal instead is to maximize the disentanglement of a GPQC.

DiVincenzo argues that a quantum system should change its own state over time by only a small amount. To describe this he uses the formula

$$\langle\psi|\rho|\psi\rangle \geq 1 - \epsilon$$

in which $\boldsymbol{\psi}$ describes the state and ρ the derived state after one clock cycle.
The parameter ϵ is dependent on the error tolerance and correction of the specific implementation of the quantum system. The current development works with factors around 10^{-6}, but it might be possible to go up to 0.1 or further in the future.

- Transformation

It is essential to transform the quantum system from one state to another over a sequence of unitary transformations carried out from outside the quantum system.
This transformation can be described as a vector in the Hilbert space describing the quantum system and is usually represented by a Hamiltonian of the form

$$U = T\exp(i\int H(t)dt)$$

These transformations can be achieved in various ways, depending on the implementation of the quantum system. Common approaches include magnetism, optical influences (e.g., lasers) and gravitational forces (easily achieveable because of the scale of quantum systems). Especially focusing these transformations on a single pair of two qubits without influencing the rest of the quantum system (quantum logic gates) is a challenge to current experimental physics.

As mentioned in the previous point, the actual constraints for errors is dependent on the actual implementation.

- Strong measurement

Strong measurement describes the accuracy of measuring the quantum state of a system.
Theoretical physics project a measuring process which completely collapses the wave function of the system and returns exactly one of the possible states as the result. In reality this has not yet been achieved on larger scales, though there exist methods for very basic quantum systems.

One example of strong measurement would be the Stern-Gerlach experiment (1922) :

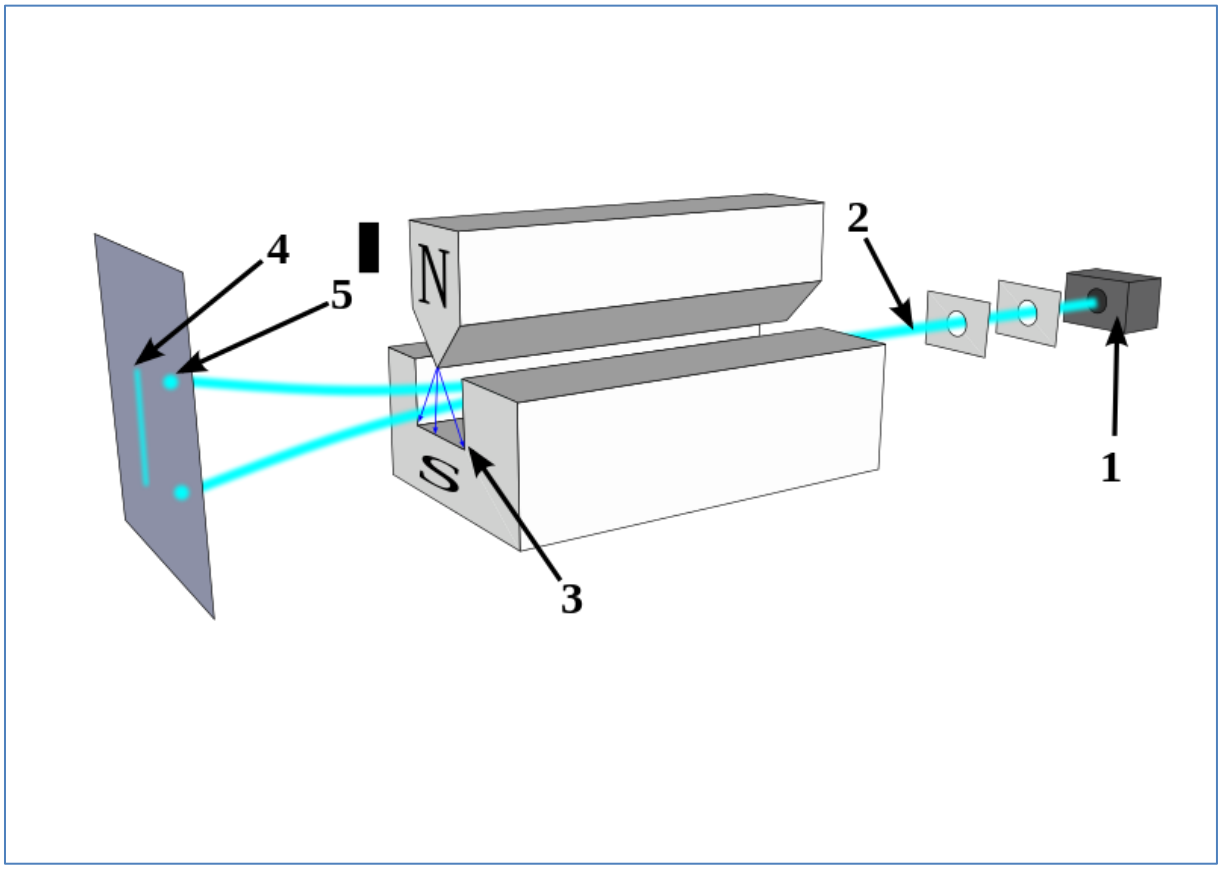

Figure 1: Stern-Gerlach experiment

In this experiment a silver atom gets deflected by an inhomogeneous magnetic field depending on its spin and then strikes a detector screen. As with the famous double-slit experiment it was expected to see a spectrum (4), though in reality the wave function collapsed and two distinct positions (5) for up- and down-spin of the silver atom respectively were visible, therefore proving the quantization of the spin of a particle.

In a simple case with exactly one particle with the states „spin up“ and „spin down“ the state is

$$\psi = a|\uparrow\rangle + b|\downarrow\rangle$$

and the strength of a measurement method for the „spin up“ case is described by

$$p_{up} = \frac{1}{2} + \delta\left(|a|^2 - \frac{1}{2}\right)$$

where δ is arbitrarily small and dependent of the implementation.
The measurements in experimental physics get validated over multiple replicas of the same system or multiple measurements on the same system. This is satisfactory for research purposes, but for a GPQC a strong measurement method would be required.

1.2. Physical and Mathematical Concepts

1.2.1. Quantum Entanglement

In general, quantum entanglement describes a multiple of particles whose quantum states can not be described independently of the others meaning that a description of the quantum state can only be made of the system as a whole. Physical properties that correlate in such a system include position, momentum, spin and polarization.

A very simple case of quantum entanglement is in a system with two photons that are known to have opposed polarizations. Such a system can be achieved by spontaneous parametric down conversion (SPDC).

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Figure 2: Type-II SPDC[1]

A photon pump sends a photon beam through a crystal which sometimes results in a pair of photons being polarized (with a probability scale of 1 in 10^{12}) . SPDC is divided into two types: Type-I and Type-II. Type-I describes a product of two photons with ordinary polarization while Type-II has a product of one photon with ordinary polarization and one with extraordinary polarization.

As shown in the figure above for Type-II SPDC each photon ends up in on of the cones, which describes the wave function of the ordinary and extraordinary photon respectively. If in this scenario the two photons are detected at the two intersections of the cones, it is not distinguishable which one is ordinary and which one is extraordinary polarized and thus a quantum superposition exists. It is now impossible to describe the polarization of one of the photons without collapsing the wavefunction of the other photon too; ergo quantum entanglement occured.

[1] http://pdxscholar.library.pdx.edu/cgi/viewcontent.cgi?article=1088&context=honorstheses, P. 15

Consequently, if the state of one of the photons is measured it is also immediately known in which state the other photon is, since it is known that the sum of states is zero. This system can be described with

$$\psi = \frac{1}{2}|H(x)V(y)\rangle + \frac{1}{2}|V(x)H(y)\rangle$$

where $H\ (horizontal) and\ V\ (vertical)$ describes the polarization of the two photons x and y.
If the state of one of the photons is measured the wave function collapses into either $H(x)V(y) or\ V(x)H(y)$ and thus the state of the other photon is also determined.

Since the two photons can be arbitrarily spatially separated and the information over the state of the other photon is availiable instantly, a paradox concerning Einstein‘s special theory of relativity arises because information is transported faster than the speed of light. Einstein himself entitled this fact as „spukhafte Fernwirkung“ commonly translated to „spooky action at a distance“ (Buhrman, Cleve, & van Dam, 2006).

The paradox goes even further when both photons are measured. Assuming there are two measuring events $m_x\ and\ m_y$ it is not possible to order these events on a time-scale in the same inertial frame while also assuming special relativity. Therefore, observers of each event would disagree on which measurement actually caused the wave function to collapse.

Einstein thought that the solution to this paradox lies in the existence of unobservable (here: limited by the human capabilities) properties that predetermine the outcome of the measurement and therefore refuting quantum mechanics. This theory is called the „hidden variables theory“.

If this theory were correct it would satisfy Bell’s inequality statistically by showing the missing correlation between the two observed particles. Experimental physics proved that Bell’s inequality is not satisfied and therefore disproved the hidden variables theory. (Christensen et al., 2013).

Quantum entanglement also induces the emergence of time. This effect is described by the Wheeler-DeWitt equation and was proven by Don Page and William Wootters. The experiment results in the non-existence of time for an external observer outside the universe (as predicted by Wheeler and DeWitt) and deduces the existence of time for an internal observer by becoming entangled with the universe. (The Physics arXiv Blog, 2013; Moreva, Brida, Gramegna, Givoannetti, Maccone, & Genovese, 2014)

1.2.2. Quantum Teleportation

In terms of communication, quantum state teleportation is sometimes generalized as quantum teleportation. It is important to differentiate between state, energy and particle teleportation.

Communication using quantum teleportation is generally achieved by shifting entanglement systems. Assume Alice and Bob both posess a qubit which are maximally entangled and therefore can be represented with one of the Bell states:

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B)$$
$$|\Phi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B)$$
$$|\psi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B)$$
$$|\psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B)$$

where $|0\rangle_A$ means that Alice‘s qubit is in state 0.

These four states basically depict the four possible combinations of any property of the respective qubit, e.g. spin or polarization. Now, Alice wants to transmit the quantum state of another qubit C to Bob.

$$|\psi\rangle_C = \alpha|0\rangle_C + \beta|1\rangle_C$$

Assuming that Alice’s and Bob’s qubits are in the state $|\Phi^+\rangle$, with the added qubit C the following state is present:

$$|\Phi^+\rangle_{AB} \otimes |\psi\rangle_C = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B) \otimes (\alpha|0\rangle_C + \beta|1\rangle_C)$$

There are again four possibilities for the combined states of A and C. This can be represented as another superposition term:

$$|\Phi^+\rangle_{AB} \otimes |\psi\rangle_C = \frac{1}{2}\begin{pmatrix} |\Phi^+\rangle_{AC} \otimes (\alpha|0\rangle_B + \beta|1\rangle_B) + |\Phi^-\rangle_{AC} \otimes (\alpha|0\rangle_B - \beta|1\rangle_B) \\ +|\psi^+\rangle_{AC} \otimes (\beta|0\rangle_B + \alpha|1\rangle_B) + |\psi^-\rangle_{AC} \otimes (\beta|0\rangle_B - \alpha|1\rangle_B) \end{pmatrix}$$

Now if Alice proceeds to make a Bell measurement of her two particles A and C, those two are forced into a Bell state and entanglement occurs while the entanglement between A and B is decomposed. The above superposition composes to one of these four states:

$$|\Phi^+\rangle_{AC} \otimes (\alpha|0\rangle_B + \beta|1\rangle_B)$$

$$|\Phi^-\rangle_{AC} \otimes (\alpha|0\rangle_B - \beta|1\rangle_B)$$

$$|\psi^+\rangle_{AC} \otimes (\beta|0\rangle_B + \alpha|1\rangle_B)$$

$$|\psi^-\rangle_{AC} \otimes (\beta|0\rangle_B - \alpha|1\rangle_B)$$

The state of Bob's particle is now dependent on the original state of C. To transform his particle B into the original state of C he needs to apply one of four transformations dependent on which Bell state Alice's two particles are in now. To get hold of that information, Alice has to send Bob two bits of classical information, representing the Bell state her particles are in. Bob can now transform his particle locally via quantum gates and ends up with having a copy of the original state of particle C.

The transfer of classical information faster than light is therefore not possible with this procedure because it requires communication of two bits of classical information over classical channels. This reduces the amount of information that has to be transferred and secures the communication. For a whole quantum state teleportation only two bits of classical communication are required, which would not be sufficient to communicate the whole quantum state. Also, an interceptor on the classical channel only receives the action that Bob needs to perform on his particle to achieve the original state, but he does not gain any knowledge on the particle state itself.

1.2.3. Quantum Annealing

One use case and huge superiority quantum computers have over classical computers is finding the minimum of cost functions with a massive speedup. The difference to the classical simulated annealing is the utilizing of quantum tunneling as replacement for thermal activation (sometimes called thermal jumps).

Assume a cost function represented by a physical system (a usual representation is finding the ground state of a spin glass). If defined over a physical system, the function values are defined over various energy states and therefore finding the minimal energy state corresponds to finding the minimal function value.

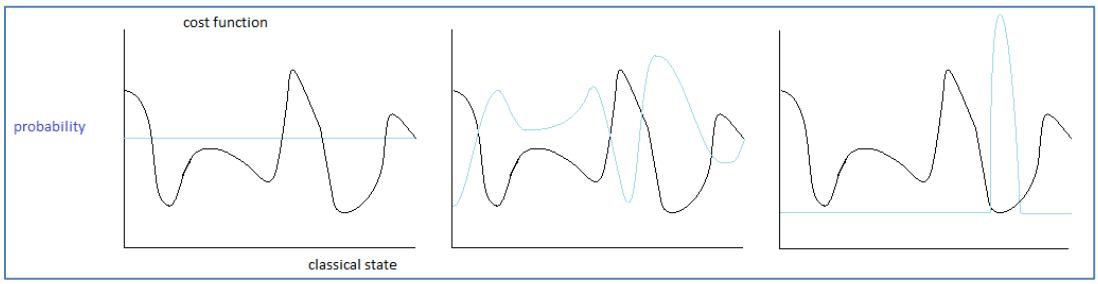

Figure 3: Solving a cost function with quantum annealing

The idea of quantum annealing is to represent the cost function as an energy function and then use a quantized particle to find the global minimum. For that, the energy state of the particle gets lowered until it is almost zero. Since quantized particles can cross small borders through quantum tunneling, it is not getting stuck in local minima in the process. With simulated annealing a particle would have to be lifted over borders between local minima via thermal activation, i.e. inserting energy into the system again.

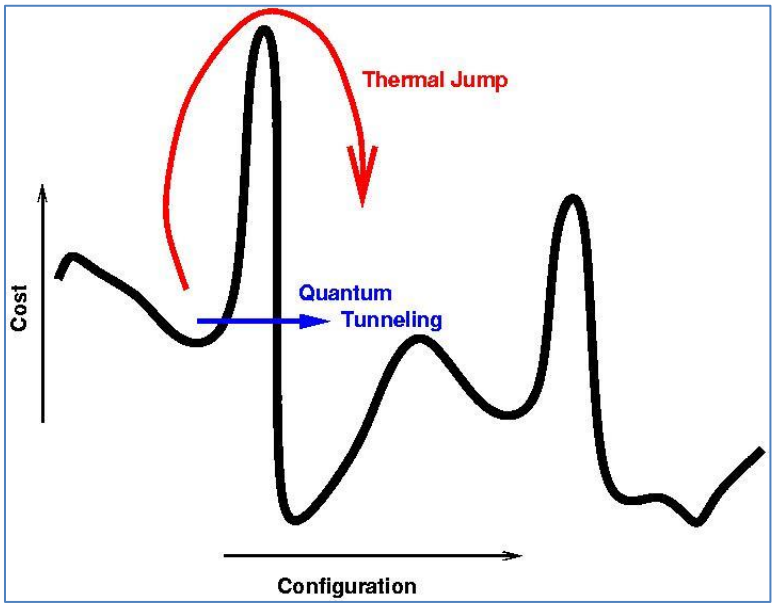

Figure 4: Quantum tunneling vs. thermal jump in annealing process

1.2.3.1. Quantum Tunneling

Quantum tunneling describes the phenomenom of a particle crossing a barrier that it would not be able to pass by in classical physics. The phenomenom is often explained via the Heisenberg uncertainty principle. Earle Hesse Kennard and Hermann Weyl formulated this principle into the form

$$\sigma_x \sigma_p \geq \frac{\hbar}{2} \; where \; \hbar = \frac{h}{2\pi}$$

$$\sigma_x \; is \; the \; standard \; derivation \; of \; position \; and \; \sigma_p \; is \; the \; standard \; derivation \; of \; momentum$$

This formula implies that neither σ_x nor σ_p can be zero. What this means is that there is no momentum and no position that a particle can not inherit. In other words: there is always a small probability that a particle is 'across' the barrier it is passing through even if there is no classical explanation for this effect.

In modern technology this effect has already been established and used, for example, in flash memory to erase NOR flash cells.

1.3. Potential

1.3.1. Cryptography

Quantum computing posesses the potential for powerful new ways of cryptography. This ranges from perfectly random key generation and secure transmitting to new crypto-algorithms that are only usable by quantum systems.

It also poses a threat for already existing cryptography by moving problems like integer factorization into the class of BQP-complex problems (originally estimated to be NP-complete). Algorithms utilizing this have been developed long before physical quantum computers have become reality. An example for such an algorithm is Shor's algorithm which (if the given hardware implementation for arbitrary sizes would exist) solves integer factorization in

$$O((log\ N)2(log\ log\ N)(log\ log\ log\ N))$$

More detailed information on the topic of quantum cryptography can be found in the paper of Céline Nöldemann; more information on attacks on existing cryptography mechanisms in the papers of Eduard Zeller and Björn Philip Ridder. All papers mentioned above have been published in this same course and will be published on the webpage of Prof. Dr. Gerd Beuster.[2]

[2] http://www.fh-wedel.de/mitarbeiter/gb

1.3.2. Popular Superstition

1.3.2.1. Faster-than-light

The fact that (as described in 1.2.2.) the information exchange in Alice's and Bob's quantum system is instantaneous may create the impression that faster-than-light classical information exchange is possible. This would disprove Einstein's hypotheses of relativity that the speed of light

$$c = 299792.458 \ {}^{m}/_{s}$$

is the fastest possible speed for any information.

Commonly the theory of faster-than-light information exchange is seen to be wrong and refuted by the no-communication theorem. The implication that is made is that Bob can have no knowledge of what Alice did to her part of the quantum system and therefore no information can be exchanged.

The no-communcation theorem is based on multiple other rules and theorems, which are all neither proven nor disproven yet. Therefore, it is currently assumed that faster-than-light communication is not possible to maintain the implications made by relativity and causality.
These theorems include the no-cloning theorem.

1.3.2.2. Physical teleportation

Quantum teleportation sometimes leads to the false impression of teleportation in a physical sense as depicted in various science-fiction works. As mentioned above, the currently usual meaning of the term quantum teleportation describes quantum state teleportation and therefore no exchange of matter, but only of state information.

Nonetheless, Yuchuan Wie drafted a paper on the the actual physical teleportation of single particles as an explanation for, e.g. superconductivity. This concept has neither been verified nor debunked as of yet (Wei, 2016).

1.4. Comparison to classical computers

In many aspects, quantum computers are similar to classical computers. The biggest difference is the usage of quantum effects for complex mathematical operations and the probabilistic nature of current implementations.

The smallest unit of storage in quantum computers is called a qubit and depicts a quantum system in a superposition over exactly two states. Current developments use varying implementations such as photons, electrons, whole atoms or even molecules to encode this quantum system.
Multiple qubits can be combined to storage units such as registers, but the process to implement this is rising exponential in difficulty with the amount of qubits. This is because all qubits have to be entangled with each other, creating a superposition over all possible combined states.

Basic operations on qubits are usually depicted by a concept of gates, very similar to the concept of gates in classical computing. These gates are defined by their underlying transformation matrix and their implementation and effectiveness therefore varies. Some gates have identical semantics to classical gates such as the NOT-gate while others are completly unique to the quantum computing world, for example the Ising gate.

As stated above, current implementations of quantum computers work on a probabilistic nature due to the nature of lacking measurement- and implementation precision. Compared to DiVincenzos studies, most current implementations would not fulfill the requirements to scale into a GPQC.

Some implementations combine classical computing with quantum computing by using classical computing for basic tasks such as loops and quantum computing operations for any parts that benefit from the increased speed and aren’t as vulnerable to probabilistic errors.

2. Development

2.1. D-Wave systems

Founded in 1999, D-Wave Systems Inc. quickly became one of the leading forces in the development of quantum systems and general purpose quantum computers. The company works together with powerful marketforces like Lookheed Martin and Google, as well as government- or international groups such as NASA and USRA and various academic institutions.

2.1.1. Achievements

The first public demonstration of a quantum computer happened in 2007 in California where D-Wave demonstrated their Orion system, a 16-qubit processor aimed at solving specific NP-complete problems opposing to depicting a general purpose quantum computer.

In 2011 D-Wave published their next system, the D-Wave One. They ramped up the qubit-amount to 128 and claimed it to be the first commercially available quantum processor. The D-Wave One utilizes quantum annealing for solving optimization problems.

The development continued with the D-Wave Two (512 qubits, released in 2012), the D-Wave 2X (2048 qubits with 1152 qubits enabled, released in 2015) and the D-Wave 2000Q (2048 qubits, released in early 2017).

2.1.2. Criticism

D-Wave Systems received a lot of negative feedback concerning the speedup of their quantum processors. Multiple experiments have disproven a speedup or even shown classical computation to be faster (Rønnow, et al., 2014; Selby, 2013).

Umesh Vazirani, specializing on quantum complexity theory, criticized D-Wave with the following words:

„Their claimed speedup over classical algorithms appears to be based on a misunderstanding of a paper my colleagues van Dam, Mosca and I wrote on "The power of adiabatic quantum computing." That speed up unfortunately does not hold in the setting at hand, and therefore D-Wave's "quantum computer" even if it turns out to be a true quantum computer, and even if it can be scaled to thousands of qubits, would likely not be more powerful than a cell phone.“ (Vazirani, 2007)

It is therefore to be seen very critical if the progress made by D-Wave Systems is actual progress into the field of quantum computing or just enhancement of classical computing with quantum mechanics for small subproblems. Another factor of criticism is the large amount of qubits they achieved in a short time, while other companies such as IBM just developed a 16-qubit GPQC (IBM, 2017).

3. Glossary

- Hilbert Space

An abstract vector space with any finite or infinite number of dimensions. It is a complete vector space using the scalar product determining the length of vectors as a norm and therefore being a special version of a banach space.

- Banach Space

A description for a complete and normed vector space. It uses a metric (or: norm) to allow computation of vector lengths.

- Vector Space

A collection of vectors over a given field that fulfills certain axioms (axioms of real vector spaces).
If a norm exists that defines a function to assign a positive length or size to each vector in this vector space, it is called a normed vector space.
A vector space is called complete when every Cauchy sequence of vectors in this vector space has a limit in this vector space or converges in this vector space.

An example for an incomplete metric space would be the rational numbers. $\sqrt{2}$ is not an element of the rational numbers, yet there exists a sequence over the rational numbers with

$$x_1 = 1 \; and \; x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n}$$

which converges to $\sqrt{2}$.

- Cauchy Sequence

A numerical sequence „whose elements become arbitrarily close to each other as the sequence progresses.“ (Lang, 1993)

- Hamiltonian

An operator (usually denoted as H,Ȟ or Ĥ) that describes the energy state of a quantum system.
It is commonly used to represent the state of quantum system in a time-dependent fashion such as

$$\int H(t)dt$$

- Spin

Describes the rotational momentum of a particle around itself; the particle spins around its own axis.
Experiments like the Stern-Gerlach experiment prove the quantization of the spin of an atom.

- Ordinary and Extraordinary Polarization

Also called parallel and perpendicular polarization; the polarization of light changing through an optical medium along an optic axis.
Parallel or ordinary polarization preserves the orientation along the axis while perpendicular or extraordinary polarization tilts the orientation (from horizontal to vertical or vice versa).

- Bell's inequality or Bell's theorem

The simplest form of Bell's theorem is: *„No physical theory of local hidden variables can ever reproduce all of the predictions of quantum mechanics.*"
To prove this point, John Stewart Bell defined an inequality which would be true if his theorem were false. The prove uses measurements of two parties on two entangled particles. Each party possess two analyzers with three arbitrary settings each for measuring a property of the given particle. The inequality makes a statistical statement over the outcome of such measurements. Experiments have found differing results in a sufficient amount, disproving the inequality and therefore proving Bell's theorem.

- Bell measurement

A measurement concept to measure two qubits simultaneously, determining them to be in one of four so-called Bell states. Since the measurement defines a new quantum system over those two qubits, they become entangled if they weren't entangled before.

- Spin Glass

A special kind of magnet in which the individual components are disaligned to each other (i.e. not all oriented in the same direction). Usually it is stuck in a superposition over stable states without being able to reach the minimal energy stable state (i.e. all components oriented in the same direction). This inability is called frustration.

- Complexity Classes

A system for classification of the difficulty of computational problems. Famous examples are the P- and NP-complete-class.

- BQP problems

Short for bounded-error quantum polynomial time. A complexity class including problems that can be solved by a quantum computer in polynomial time with an error of at most $\frac{1}{3}$. It is currently unknown what the relationship between BQP- and NP-problems is.

- No-cloning Theorem

A theory claiming that it is not possible to copy the exact quantum state of an arbitrary quantum system. The current proofs for this theory base on other theories, so it is not yet definetly known if this theorem is true.

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Figures

Figure 1: Stern-Gerlach experiment
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Figure 2: Type-II SPDC
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Figure 3: Solving a cost function with quantum annealing
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Figure 4: Quantum tunneling vs. thermal jump in annealing process
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